Ten Ways for Conditionals to Express Conditional Belief

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• Conditional content totally penetrates ordinary language:
  – causation (which is ubiquitous in turn, hides, e.g., in transitive verbs),
  – dispositions, laws, etc. etc.

• The linguistic phenomenology, however, is a mess:
  – not only “if, then“, but also “even if“, “although“, “because“, and many more,
  – interaction with tenses, moods, and aspect,
  – scope ambiguities with negation, quantifiers, and modals,
  – complex pragmatics,
  – presumably not grammatically identifiable at all (“you scratch my back, and I scratch yours”).
A Fresh Start is Needed (2)

• The philosophical situation is a mess as well:
  – What at all is conditional content?
  – Maybe there are various conditional relations (as suggested by the Oswald-Kennedy example).
  – At least there are many different accounts of conditional relations, often at odds.
  – Problems with relevance (of the antecedent for the consequent).
  – Problems with truth-conditions for conditionals (which should not be entirely denied, but not assigned too easily and extensively).
  – Problems with the iterability of conditionals.

• The philosophical and the linguistic problems are intertwined. The linguistic arena is probably the proof of the philosophical accounts. However, how to check one mess with the help of another?
Expressivism (1)

• So, perhaps we should start from scratch. I propose to start with expressivism.
• What is language good for? Primarily for expressing our mental states. This seems obvious. At least, our mental states are the direct causal predecessors of our speech; so, whatever else it signifies is mediated by them. This idea favors an expressivist strategy for doing semantics.
• Why, then, is truth-conditional semantics the favored strategy instead?
  – Because we know how to do semantic recursion in terms of truth-conditions;
  – because we usually express propositional attitudes, in particular beliefs, the content of which are propositions, i.e., truth-conditions;
  – because mental states and their contents are identifiable only with reference to external states of affairs.
Expressivism (2)

- However, truth-conditional semantics is not good enough for all mental states we might wish to express. At least, I am very sure that conditional belief escapes this strategy.

- What is conditional belief? In any case something most important for our cognitive life, since it governs the dynamics of belief.

- And: conditional belief is not a propositional attitude; it is a bi-propositional attitude. It relates two propositions, i.e., truth-conditions, but the relation, in turn, cannot be grasped in truth-conditional terms. Conditional belief does not reduce to belief in a proposition. So, truth-conditional semantics must fail. This fundamental insight is crucial.

- How do we express conditional belief? As mentioned above, in multifarious ways. Reversely, however, it seems clear that the conditional idiom is essentially designed for expressing conditional belief.

- The long and the short of all this: Any investigation of conditionals must proceed from a study of conditional belief.
Conditional Belief (1)

- This insight goes back to Ramsey, who, however, did not yet have a suitable theory of conditional belief. What might it be?
- **Not probability theory.** It provides an adequate account of conditionality, but not of belief; probability is not belief. So, Adams’ admirable strategy is at best a round-about way of approaching conditionals.
- **Not AGM belief revision theory,** which offers no complete dynamics of belief and hence no adequate notion of conditional belief.
- **Not Stalnaker-Lewis conditional logic,** which is not about conditional belief, anyway, the doxastic reinterpretation of which, however, would amount to AGM and is hence formally insufficient.
- The inadequacy of the latter two theories also shows in their inability to provide an adequate account of relevance and irrelevance.
- In my view, the only adequate account of conditional belief is provided by ranking theory. Thus, if expressivism is correct, any theory of conditionals must be based on ranking theory and nothing else.
Conditional Belief (2)

- What is ranking theory?
- Let $W$ be a non-empty set of possible worlds, $\mathcal{A}$ be an algebra of propositions over $W$, $N$ be some range of numbers, and $\infty$ some number larger than any in $N$.
- $\kappa$ is a conditional negative ranking function iff $\kappa$ is a function from $\mathcal{A} \times \mathcal{A} - \{\emptyset\}$ into $N^+ = N \cup \{\infty\}$ such that for all $A, B, C \in \mathcal{A}$ with $A \neq \emptyset$:
  1. $\kappa(\neg A | A) = \infty$,
  2. $\min \{\kappa(B | A), \kappa(\neg B | A)\} = 0$,
  3. $\kappa(B \cap C | A) = \kappa(B | A) + \kappa(C | A \cap B)$, provided that $A \cap B \neq \emptyset$.
- Unconditional negative ranks are then defined as $\kappa(B) = \kappa(B | W)$.
- (This is a variant of ranking theory with conditional ranks as primitive notion, essentially equivalent to the standard version and perhaps even more convincing.)
Conditional Belief (3)

• Negative ranks represent degrees of disbelief. That is, $\kappa(B \mid A)$ is the degree of disbelief in $B$ given or conditional on $A$. Thus, $\kappa(B \mid A) = 0$ says that $B$ is not disbelieved given $A$, and $\kappa(B \mid A) > 0$ says that $B$ is disbelieved (to some degree) given $A$. Hence, that $B$ is believed given $A$ is represented by $\kappa(\neg B \mid A) > 0$.

• From a conditional negative ranking function $\kappa$ we may define a conditional positive ranking function $\beta$ by: $\beta(B \mid A) = \kappa(\neg B \mid A)$. Such a function represents conditional belief directly.

• For $A = W$ axiom (3) translates into $\beta(B \rightarrow C) = \beta(\neg B) + \beta(C \mid B)$ (with $\rightarrow$ as material implication or its set-theoretical counterpart). This is the true relation between conditional belief and belief in a material implication.

• A further notion derived from $\kappa$ and/or $\beta$ is the conditional two-sided ranking function $\tau$ defined by: $\tau(B \mid A) = \beta(B \mid A) - \kappa(B \mid A)$. Two-sided ranks express belief and disbelief in one function.
Ranking-theoretic semantics for standard non-iterated conditionals (1)

Ranking functions immediately help us to a semantics for a non-iterated fragment $L_1$ of conditional logic implementing the Ramsey test. The syntax is simple: Let $L_0$ be the language of propositional logic, and let $\succ$ stand for the conditional. Then, if $\phi$ and $\psi$ are sentences of $L_0$, $\phi \succ \psi$ is a sentence of $L_1$, and if $\phi$ and $\psi$ are sentences of $L_0$ or $L_1$, propositional combinations of $\phi$ and $\psi$ are sentences of $L_1$, too. Thus, no nestings of $\succ$ can occur in $L_1$.

As for the semantics, let $V$ be the set of valuations (of the sentence letters) of $L_0$; and for each $\phi \in L_0$, let $V(\phi)$ be the set of valuations in which $\phi$ is true. Moreover, for any ranking function $\kappa$ for $V$, let $\mathcal{B}(\kappa) = \{\phi \mid \tau(V(\phi)) > 0\}$ the set of beliefs in $\kappa$ (where $\tau$ is the associated two-sided ranking function); let $\mathcal{CB}(\kappa) = \{\phi \succ \psi \mid \tau(V(\psi) \mid V(\phi)) > 0\}$ be the set of conditional beliefs in $\kappa$; and for $v \in V$, let $T(v) = \{\phi \in L_0 \mid \phi$ is true in $v\}$. Finally, let $|=_{PL}$ denote validity in propositional logic.
Now we have a choice. According to a purely epistemic semantics, we may define that $\varphi \in L_1$ is true according to a ranking function $\kappa$ for $V$, $\kappa \models e \varphi$, iff $\mathcal{B}(\kappa) \cup \mathcal{CB}(\kappa) \models_{PL} \varphi$, and logically true, $\models e \varphi$, iff $\kappa \models e \varphi$ for all ranking functions $\kappa$ for $V$. Or we may establish a semi-epistemic semantics by defining that $\varphi \in L_1$ is true according to a valuation $v \in V$ and a ranking function $\kappa$ for $V$, $\langle v, \kappa \rangle \models_{se} \varphi$, iff $T(v) \cup \mathcal{CB}(\kappa) \models_{PL} \varphi$, and logically true, $\models_{se} \varphi$, iff $\langle v, \kappa \rangle \models e \varphi$ for all valuations $v \in V$ and ranking functions $\kappa$ for $V$.

The restriction of Lewis’ logic $\mathbf{VC}$ to the fragment $L_1$ is correct and complete with respect to $\models e$. In particular, Weak Centering holds, since $\tau(B \mid A) > 0$ entails $\tau(A \rightarrow B) > 0$, and Centering holds as well, since $\tau(A \cap B) > 0$ entails $\tau(B \mid A) > 0$. However, according to $\models e$ these axioms only indicate a relation between conditional and unconditional belief.

By contrast, it is only Lewis’ logic $\mathbf{V}$, restricted to the fragment $L_1$, that is correct and complete with respect to $\models_{se}$; in particular, neither Centering nor Weak Centering hold with respect to $\models_{se}$, because there is no relation between the facts according to $v$ and the conditional beliefs according to a ranking function $\kappa$ for $V$. 

Ranking-theoretic semantics (2)
What Conditionals Might Express (1)

• My subsequent strategy is this: Don’t try to directly account for the linguistic phenomenology. Then you are likely to drown in the mess.
• Rather, if the conditional idiom is for expressing conditional belief, then give a survey of all we might want to express about conditional belief. It is easier to succeed in this attempt, because we have a complete theory of conditional belief that is much simpler than the conditional idiom. And if we succeed, we have provided everything for interpreting the conditional idiom (even though I won’t actually engage in that interpretation).
• A main point of my talk will be to convince you that our expressive options are much richer than we are used to think.
What Conditionals Might Express (2)

- So, what is it we might want to express? From now on let me use the symbol $\triangleright$ for generically representing any conditional we might use ("if, then", "even if", "although", because", etc.).
- The only idea discussed in the literature is to interpret $A \triangleright B$ as expressing the Ramsey test (on which the semantics above was based as well)
  (I) (a) $\tau(B | A) > 0$, i.e., the conditional belief in $B$ given $A$.
  (I) (b) $\tau(B | A) = 0$, or (c) $\tau(B | A) < 0$.
- We might use $A \triangleright B$ also for expressing or combinations thereof. For instance, might-conditionals are usually taken as expressing (Ia)-or-(Ib).
What Conditionals Might Express (3)

• However, in asserting $A \supset B$ there are many more attitudes towards $A$ and $B$ and their relation we might wish to express than explained by the Ramsey test. Let’s not forget that we might also express our attitude towards $A$ and towards $B$ by itself, i.e., whether

(II) (a) $\tau(A) > 0$, (b) $\tau(A) = 0$, or (c) $\tau(A) < 0$, and whether

(III) (a) $\tau(B) > 0$, (b) $\tau(B) = 0$, or (c) $\tau(B) < 0$,

i.e., whether we take the antecedent and the consequent of the conditional to be true or false.

• We cannot only express belief in $A$, etc., but also strength of belief in $A$. There are many modifiers in natural language indicating strength of belief, at least roughly and vaguely, and they are often used in conditional constructions. However, since they are not specific to those constructions, I shall not further dwell upon them.
What Conditionals Might Express (4)

- More interesting is, of course, what we might express about the relation of the antecedent and the consequent. There, the Ramsey test (Ia) is only a first idea. Another important idea is relevance. That is, we might use $A \succ B$ for expressing the positive or negative relevance or the irrelevance of $A$ for $B$, i.e.:

  (IV) (a) $\tau(B \mid A) > \tau(B \mid \neg A)$, (b) $\tau(B \mid A) = \tau(B \mid \neg A)$, or
  (c) $\tau(B \mid A) < \tau(B \mid \neg A)$.

- Note that an adequate representation of relevance as in (IV) builds on the full resources of ranking theory; entrenchment orderings or similarity spheres won’t do for establishing the comparisons required by (IV).
What Conditionals Might Express (5)

• A is a reason for $B$ just if $A$ speaks for $B$, i.e., if $A$ is positively relevant to $B$. Now, there are four different kinds of reasons or positive relevance, (IVa) differentiates accordingly, and we might use $A \triangleright B$ also for expressing those kinds:

(V)  
(a) $\tau(B \mid A) > \tau(B \mid \neg A) > 0$, i.e., $A$ is a supererogatory reason for $B$,  
(b) $\tau(B \mid A) > 0 \geq \tau(B \mid \neg A)$, i.e., $A$ is a sufficient reason for $B$,  
(c) $\tau(B \mid A) \geq 0 > \tau(B \mid \neg A)$, i.e., $A$ is a necessary reason for $B$,  
(d) $0 > \tau(B \mid A) > \tau(B \mid \neg A)$, i.e., $A$ is an insufficient reason for $B$.

• Note that $A$ may be a necessary and sufficient reason for $B$.

• Note also that the variants of (II) – (IV) could be combined freely so far. This changes with (V). There are a lot of interesting interactions between (V) and (II) and (III).
What Else Conditionals Might Express (1)

• It might appear that (I) – (V) exhaust our doxastic or epistemic attitudes towards two propositions $A$ and $B$ and their relation. However, there is a further class of beliefs we might express with conditionals, which is most important, and often suggested, though not explicitly treated in the literature.

• Let $\mathcal{P}$ be a partition of $W$. Then we may use the conditional $A \triangleright B$ for expressing our belief that some condition in $\mathcal{P}$ obtains under which we believe $B$ given $A$, or under which we take $A$ to be positively relevant to $B$ – formally:

(VI) $\tau(C^*) > 0$, where $C^* = \bigcup P^*$ and $P^* = \{C \in \mathcal{P} \mid \tau(B \mid A \cap C) > 0\}$, and

(VII) $\tau(C^*) > 0$, where $C^* = \bigcup P^*$ and $P^* = \{C \in \mathcal{P} \mid \tau(B \mid A \cap C) > \tau(B \mid \neg A \cap C)\}$. 
What Else Conditionals Might Express (2)

• We might differentiate (VII) in the same way as did (V) with (IV); this generates the expressive possibilities (VIIa) – (VIIId).

• Where does the partition $\mathcal{P}$ come from? The general answer is that it is somehow contextually given, but later on I shall discuss a particularly salient instantiation of $\mathcal{P}$ that is almost always referred to.

• For an example of (VI) and (VII) look at Quine’s famous pair concerning the Korean war:
  – “If Cesar were in command, he would use the atomic bomb.”
  – “If Cesar were in command, he would use catapults.”

• The crucial point about (VI) and (VII) is that according to them conditionals express unconditional beliefs, which are true or false in an unproblematic way and about which one can have a factual argument. So, this accounts for the intuition that conditionals are (also) about facts.
What Else Conditionals Might Express (3)

• If atomic propositions refer to certain times, and if \( A_{t'} \) and \( B_t \) are such atomic propositions referring, respectively, to \( t' \) and \( t \), then there is a natural, context-independent partition associated with the conditional \( A_{t'} \triangleright B_t \), namely the partition \( \mathcal{H}_{t',t} \) of all histories up to \( t \) without \( t \) and \( t' \) (as far as specifiable within the given algebra).

• Then (VI) and (VII) specialize thus: By asserting \( A_{t'} \triangleright B_t \) I can express my belief that history is such that \( B_t \) must obtain given \( A_{t'} \), or that history is such that \( A_{t'} \) is positively relevant to \( B_t \), i.e.:

\[(IX) \quad \tau(H^*) > 0, \text{ where } H^* = \bigcup \mathcal{H}^* \text{ and } \mathcal{H}^* = \{H \in \mathcal{H}_{t',t} \mid \tau(B_t \mid A_{t'} \cap H) > 0\},\]

\[(X) \quad \tau(H^*) > 0, \text{ where } H^* = \bigcup \mathcal{H}^* \text{ and } \mathcal{H}^* = \{H \in \mathcal{H}_{t',t} \mid \tau(B_t \mid A_{t'} \cap H) > \tau(B_t \mid \neg A_{t'} \cap H)\}.\]

• I shall refer to this reading as the “circumstances are such that” or “history is such that” reading of conditionals.

• We might differentiate (X) in the same way as (VII).
What Else Conditionals Might Express (4)

Various things are remarkable about (IX) and (X):

1) The relevant partition is given by the conditional $A_{t'} \triangleright B_t$ itself and not by the context.

2) This conditional expresses an unconditional belief about the history of $B_t$.

3) Which unconditional belief is expressed depends on the ranking function $\tau$. However, according to (IX) and (X) it depends only on a special part of it, which might be called its predictive strategy, i.e., only on conditional beliefs about the immediate future given the entire history.

4) The predictive strategy embodied in a ranking function is particularly stable, insofar as it cannot change through any kind of historical information; only the actual predictions can change thereby.

5) So, one might hope for intersubjective agreement on the predictive strategy. Then we find so much agreement also concerning the unconditional belief expressed.
What Else Conditionals Might Express (5)

(6) We may express (IX) and (X) in more suggestive terms:
   - (IX) expresses the belief that (history is such that) $B_t$ is determined by its past (in the old Aristotelian sense of determination).
   - (X) expresses the belief that (history is such) that $A_t$ is a direct cause of $B_t$.

(7) The latter claim is based on the analysis of causation I am propagating for a long time.

(8) In this way, (X) can account for our deeply entrenched causal reading of conditionals.

(9) The proposition $H^*$ in (IX) and (X) (or $C^*$ in (VI) and (VII)) represents what many authors call the base of a conditional: the base on which it is asserted, which is supposed to consist in the facts that make the conditional true or assertible (Edgington) or on which the conditional supervenes (Lewis).
Truth Conditions for Conditionals? (1)

- So far I have offered ca. 10 options for what we might express with conditionals, which go far beyond the Ramsey test: they add the idea of relevance, the add the “circumstances are such that” reading, and they differentiate. All these extensions presuppose full ranking theory.

- I have no proof that my list exhausts all expressive options. Still, I guess the list is a rich offer to the linguist to account for the confusing linguistic phenomenology.

- For instance, take the famous Oswald-Kennedy pair. If the indicative conditional is interpreted according to the Ramsey test and the subjunctive according to the “history is such that” reading, they express, and are assertible, by a perfectly coherent epistemic state, i.e., ranking function.
Truth Conditions for Conditionals? (2)

- Still, the fundamental question remains: May conditionals be called true or false and thus have truth conditions? So far not; they just express epistemic states, in particular conditional beliefs.
- However, my account provides some systematic positive offers. Here are my results in a summary fashion:
- Ranking theory comes along with an objectivization theory telling which features of ranking functions are objectivizable, i.e., can be uniquely associated with propositions that are objectively true or false. According to this objectivization theory:
  - Unconditional belief is objectivizable (since each belief uniquely corresponds to the proposition believed and thus has a truth condition).
  - Conditional belief is generally not objectivizable.
  - Positive relevance is generally not objectivizable.
Truth Conditions for Conditionals? (3)

(1) Hence, Ramsey test conditionals (Ia) are generally not objectivizable and can generally not be assigned truth conditions. (This is no surprise after Lewis’ and AGM trivialization theorems.)

(2) Likewise, relevance conditionals (IV) and (V) can generally not be assigned truth conditions.

(3) Insofar conditionals express (dis-)belief in the antecedent and the consequent according to (II) and (III), they are objectivizable.

(4) The “circumstances are such that” conditionals (VI) and (VII) are objectivizable and thus have truth conditions, since they express unconditional beliefs (circumstances actually are or are not the way required). However, the truth condition is context-dependent like the relevant partition referred to. And it still depends on the underlying ranking function, because which belief is expressed depends on this function. If this function is known or agreed, dispute about such conditionals is purely factual dispute.
Truth Conditions for Conditionals? (4)

(5) Observation (4) carries over to “history is such that” conditionals (IX) and (X), with two improvements: first, the context-dependence of the relevant partition vanishes, and second, intersubjective agreement on the predictive strategy, the part of the ranking function required for determining the belief expressed according to (IX) and (X), may be more easily achieved.

(6) The situation further improves, because the general non-objectivizability of conditional beliefs does not carry over to the conditional beliefs of the special form contained in a predictive strategy and referred to in (IX) and (X), which are objectivizable under certain assumptions.

(7) Hence, conditionals (IX) and causal conditionals (X) are even fully objectivizable and thus do have truth conditions under those assumptions.
Truth Conditions for Conditionals? (5)

• In my view, such a differentiated account of the truth-evaluability of conditionals is precisely what is needed; no general acceptance or general rejection of truth conditions for conditionals will do.

• Also, my expressivist strategy preserves a certain unity of the treatment of conditionals (even though it ramifies into ca. 10 expressive options). It is not badly mixed as, e.g., treating indicative conditionals as material implications and subjunctive ones in Stalnaker-Lewis style.

• Finally, my account opens some small space for the iteration of conditionals (beyond the generally accepted import-export principles), as seems intuitively the right measure.
Thank you very much for your attention!